

Comment on "Spin-Glass Solution of the Double-Exchange Model in Infinite Dimensions"

Eugene Kogan¹ and Mark Auslender²

¹ Jack and Pearl Resnick Institute of Advanced Technology,
Department of Physics, Bar-Ilan University, Ramat-Gan 52900, Israel

² Department of Electrical and Computer Engineering,
Ben-Gurion University of the Negev, P.O.B. 653, Beer-Sheva, 84105 Israel

(Dated: February 6, 2008)

PACS numbers: 75.47.Lx, 71.27.+a, 71.30.+h

In a very interesting paper submitted to the ArXiv some time ago [1], R. S. Fishman, F. Popescu, G. Alvarez, T. Maier and J. Moreno considered double-exchange model on a Bethe lattice in infinite dimensions using dynamical mean-field theory. They analyzed instabilities of the paramagnetic (PM) state with respect to infinitesimal short-range magnetic order: for any site, a fraction $1 - q$ of the neighboring spins are the same and q are opposite. (In limiting cases $q = 0$ and $q = 1$ the order is ferromagnetic and antiferromagnetic respectively.) In this, mostly pedagogical note, we present a somewhat different derivation of their general formula for the magnetic disorder - order transition temperature and comment on the interpretation of the intermediate phase obtained by the authors.

The DE model, containing classical core spins and the conduction electrons with the exchange coupling between them is a single electron one, and the Hamiltonian can be presented as

$$\hat{H}_{nn'} = t_{n-n'} - J \mathbf{m}_n \cdot \hat{\sigma} \delta_{nn'}, \quad (1)$$

where t is the electron hopping, J is the exchange coupling $\hat{\sigma}$ is the vector of the Pauli matrices, and \mathbf{m}_n is a vector of unit length which represent a core spin in a classical model.

On the Bethe lattice (Caley tree) the local Green's function

$$\hat{g}(E) = (E - \hat{H})_{nn}^{-1} \quad (2)$$

satisfies equation

$$\hat{g}_n = \frac{1}{E - \frac{W^2}{4} \hat{g}_{n+1} + J \mathbf{m}_n \cdot \hat{\sigma}}, \quad (3)$$

where W is the bare (in the absence of exchange interaction) band width. Further on all the energies we'll measure in the units of W .

In the framework of the DMFA [2] we substitute for Eq. (3) the following equation

$$\hat{G}_n = \left\langle \frac{1}{E - \hat{G}_{n+1}/4 + J \mathbf{m}_n \cdot \hat{\sigma}} \right\rangle, \quad (4)$$

where $G_n = \langle g_n \rangle$, $\langle X(\mathbf{m}) \rangle \equiv \int X(\mathbf{m}) P(\mathbf{m})$, and $P(\mathbf{m})$ is a probability of a given core-spin orientation (one-site

probability). The quantities \hat{G} are 2×2 matrices in spin space. In the PM phase $P(\mathbf{m}) = 1$ (up to normalization), $G_n = G$, and Eq. (4) is closed and takes the form

$$G = \frac{1}{2} \sum_{(\pm)} \frac{1}{E - G/4 \pm J}. \quad (5)$$

In the magnetically ordered phase the probability $P(\mathbf{m})$ should be determined self-consistently with the solution of Eq. (4). The DMFA approximation for the one-site probability $P(\mathbf{m})$ is:

$$P(\mathbf{m}) \propto \exp[-\beta \Delta \Omega(\mathbf{m})], \quad (6)$$

where

$$\Delta \Omega(\mathbf{m}) = \int_{-\infty}^{\mu} \Delta D(E, \mathbf{m}) dE, \quad (7)$$

and

$$\Delta D(E, \mathbf{m}) = -\frac{1}{\pi} \text{Im} \ln \det \left[E - \hat{G}_{n+1}/4 + J \mathbf{m}_n \hat{\sigma} \right]; \quad (8)$$

the argument of both G_{loc} and Σ is $E + i0$, and the electron gas is considered as degenerate.

Eqs. (4) and (6) present a complicated system of equations. However, near the Curie temperature the system can be reduced to an ordinary mean field equation. Eqs. (4) and (6) can be linearized with respect to small deviations of the locator from the isotropic PM value [3]. Looking for G_n in the form

$$G_n = G + \mathbf{B}_n \hat{\sigma}, \quad (9)$$

for the anisotropic part we obtain the following equation

$$(E - G/4) \mathbf{B}_n = \left(G/4 + \frac{J^2 G^2/6}{E - G/4} \right) \mathbf{B}_{n+1} - G J \mathbf{M}_n, \quad (10)$$

where $\mathbf{M}_n = \langle \mathbf{m}_n \rangle$. Similarly, Eq. (6) in linear approximation takes the form

$$P_n(\mathbf{m}_n) \propto \exp \left(\mathbf{m}_n \cdot \frac{\beta J}{2\pi} \int_{-\infty}^{\mu_P} \text{Im} \left[\frac{G \mathbf{B}_{n+1}}{E - G/4} \right] dE \right). \quad (11)$$

Ferromagnetic (FM) order is described by the equation $\mathbf{B}_{n+1} = \mathbf{B}_n$ ($\mathbf{M}_n = \mathbf{M}_{n+1}$) and antiferromagnetic (AFM) one by the equation $\mathbf{B}_{n+1} = -\mathbf{B}_n$ ($\mathbf{M}_n =$

$-\mathbf{M}_{n+1}$). Fishman et al. suggested a more general ordering, using some parameter $0 \leq q \leq 1$, described by

$$\mathbf{B}_{n+1} = (1 - 2q)\mathbf{B}_n, \quad (12)$$

so that $q = 0$ corresponds to the FM and $q = 1$ to the AFM order. After substituting this prescription into Eq. (11) we obtain an ordinary mean-field probability

$$P_n(\mathbf{m}_n) \propto \exp[-3\beta T_{cr}(q)\mathbf{M}_n \cdot \mathbf{m}_n], \quad (13)$$

where $T_{cr}(q)$ is a critical temperature given by

$$T_{cr}(q) = (1 - 2q) \frac{J^2}{6\pi} \int_{-\infty}^{\mu_P} dE \quad (14)$$

$$\text{Im} \left\{ \frac{G^2}{\left[E - \frac{G}{4}\right] \left[E - (1 - q)\frac{G}{2}\right] - (1 - 2q)\frac{J^2 G^2}{6}} \right\},$$

so that $T_{cr}(q = 0)$ is the Curie and $T_{cr}(q = 1)$ is the Neel temperature. Eq. (14) is Eq. (6) from the preprint [1], in which there was made the transition from the summation with respect to discrete frequencies to the integration with respect to energy.

Fishman et al. associated the values of $q \neq 0, 1$ with a spin glass phase. We think that the phase taken into account by the authors is a mixed state, combining both ferromagnetism and antiferromagnetism, the shell n and $n + 1$ being the sublattices. The order parameters are the vector of ferromagnetism (the averaged magnetization) $(1 - q)\mathbf{M}$ and vector of antiferromagnetism (half of the difference between the magnetization of the sublattices) $q\mathbf{M}$. From these definitions we immediately obtain Eq. (12).

-
- [1] R. S. Fishman, F. Popescu, G. Alvarez, T. Maier, and J. Moreno, cond-mat/0509270.
 [2] A. Georges, G. Kotliar, W. Krauth, and M. J. Rozenberg, Rev. Mod. Phys. **68**, 13 (1996).

- [3] E. Kogan and M. Auslender, Phys. Rev. B **67**, 132410 (2003).